Clock Bias Prediction by Application of Variational Bayesian Adaptive Kalman Filter

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***Abstract*—To improve the performance of traditional Kalman filtering algorithms in clock bias prediction, this paper proposes the application of variational Bayesian adaptive Kalman filtering algorithm to achieve clock bias prediction. This method addresses the shortcomings of traditional Kalman filters that require accurate observation noise models and system noise models in effective application. It proposes a combined algorithm of Sage Husa adaptive Kalman filtering algorithm and variational Bayesian Kalman algorithm using inverse Wishart distribution to iteratively estimate and correct observation noise *R* and system noise *Q*, estimate the covariance of system state and observation noise, and apply it to Kalman filters to improve the adaptive ability of Kalman filter clock bias prediction. Simulation experiments on filtering and clock bias prediction based on precise post ephemeris satellite clock bias data. The results show that compared with traditional Kalman filters, applying variational Bayesian adaptive Kalman filters to clock bias prediction has improved the prediction accuracy to a certain extent.**

Keywords—Variational Bayesian, Kalman Filter, Sage-Husa Filter, Clock bias prediction

I. Introduction

Clock bias prediction refers to forecasting the clock error conditions of an atomic clock for a future period based on historical clock error data. Accurate clock error prediction is vital for achieving real-time and reliable dynamic precise point positioning, accurate autonomous satellite navigation, and high-precision and effective control of atomic clock systems. Satellite clocks are a crucial component of navigation systems, and their performance and accurate prediction are paramount for ensuring the precision of navigation and positioning, as well as enabling autonomous navigation and precise positioning [1]. Ensuring their accuracy, real-time performance, and continuity is imperative. Spaceborne atomic clocks possess complex physical characteristics and are prone to interference from external factors, rendering it difficult to capture their intricate and nuanced patterns of change. The establishment of a high-precision satellite clock error prediction model has emerged as a challenging task [2]. Presently, methods employed in satellite clock error prediction encompass linear models, quadratic polynomial models, gray models, Kalman filter models, and others. However, these different single models have a limited applicable range for various precision spaceborne atomic clocks . Currently, the majority of spaceborne atomic clocks utilize rubidium atoms, and the utilization of a Kalman filter for predicting the clock error of rubidium clocks has demonstrated high prediction accuracy and robustness. The classical Kalman filter serves as an optimal estimation for linear systems adhering to a Gaussian distribution. Nevertheless, the Kalman filter with a fixed Gaussian distribution estimation exhibits certain limitations in actual satellite clock error prediction. To enhance the robustness of the Kalman filter, this paper employs a Gaussian mixture model to model non-Gaussian states or noises, superseding the traditional Gaussian distribution [3]. This paper introduces the utilization of a variational Bayes Kalman filter algorithm for satellite clock error prediction and conducts simulation experiments to validate its performance.

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II. ALGORITHM PRINCIPLE

A. Variational Bayesian Kalman filtering algorithm

The Kalman filtering algorithm believes that in the dynamic measurement process, a linear Gaussian state space model can be used to describe it. In the discrete-time linear state space model, the following equation can be established.

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{q}_{k-1} \\ \boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{r}_k \end{cases} \tag{1}$$

In (1), $\boldsymbol{x}_k$ is the n-th order state vector; $\boldsymbol{y}_k$ is the m-order measurement vector; F is the n-th order state transition matrix; $\boldsymbol{H}_k$ is the m-order observation matrix; $\boldsymbol{q}_k$ and $\boldsymbol{r}_k$ are process Gaussian white noise and measurement Gaussian white noise. Their covariance matrices are $\boldsymbol{Q}_k = E(\boldsymbol{q}_k\boldsymbol{q}_k^{\mathrm{T}})$ and $\boldsymbol{R}_k = E(\boldsymbol{r}_k\boldsymbol{r}_k^{\mathrm{T}})$, and the parameters are uncorrelated with each other at any time.

Unlike the classical Kalman filtering algorithm, the variational Bayesian Kalman filtering algorithm assumes that the state vector $\boldsymbol{x}_k$ at time k is independent of the observation noise covariance $\boldsymbol{R}_k$ .The relationship between the above parameters can be defined mathematically as follows:

$$p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid \boldsymbol{x}_{k-1}, \boldsymbol{R}_{k-1}) = p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}) \times p(\boldsymbol{R}_k \mid \boldsymbol{R}_{k-1}) \tag{2}$$

To estimate the joint posterior probability distribution $p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid y_{1:k})$, the following prediction and update steps are performed.

- Prediction process:

Assuming that at time $k-1$, the conjugate prior distribution of noise covariance is defined as the inverse Wishart distribution, the joint distribution of system state, noise covariance, and measurement is obtained, that is, the joint conditional probability distribution of $\boldsymbol{x}_{k-1}$ and $\boldsymbol{R}_{k-1}$is the product of Gaussian distribution and inverse Wishart [4]distribution，

$$\begin{aligned} p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid \boldsymbol{y}_{1:k-1}) = p(\boldsymbol{x}_k \mid y_{1:k-1})p(\boldsymbol{R}_k \mid \boldsymbol{y}_{1:k-1}) = \\ N(\boldsymbol{x}_{k-1} \mid \boldsymbol{x}_{k|k-1}, \boldsymbol{P}_{k|k-1})\boldsymbol{IW}(\boldsymbol{R}_k \mid \boldsymbol{v}_{k|k-1}, \boldsymbol{V}_{k|k-1}) \end{aligned} \tag{3}$$

where $\boldsymbol{P}_{k|k-1}$ is the error covariance of state prediction vector $\boldsymbol{x}_{k|k-1}$, and the probability density function of inverse Wishart is defined as follows,

$$\boldsymbol{IW}(\boldsymbol{R}; \boldsymbol{v}, \boldsymbol{V}) = \frac{|\boldsymbol{V}|^{\boldsymbol{v}/2}}{2^{nd/2}\Gamma_d\left(\frac{\boldsymbol{v}}{2}\right)}|\boldsymbol{R}|^{-(\boldsymbol{v}+n+1)/2}\mathrm{e}^{-\frac{1}{2}\mathrm{tr}[\boldsymbol{V}\boldsymbol{R}^{-1}]} \tag{4}$$

In (4), $\boldsymbol{V}$ is a symmetric positive definite matrix, $\boldsymbol{v}$ is a degree of freedom parameter, $\Gamma_n(\cdot)$represents a multivariate gamma function, and tr$[\cdot]$is the matrix trace.

To ensure that the conjugate prior distribution of $p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid \boldsymbol{y}_{1:k-1})$ is an inverse Wishart distribution, a change factor ρ is introduced to modify the one-step predicted values of distribution parameters $\boldsymbol{v}_{k|k-1}$and $\boldsymbol{V}_{k|k-1}$, e.g.,

$$\begin{cases} \boldsymbol{v}_{k|k-1} = \rho(\boldsymbol{v}_{k-1} - m - 1) + m + 1 \\ \boldsymbol{V}_{k|k-1} = \rho\boldsymbol{V}_{k-1} \end{cases} . \tag{5}$$

and ρ is the variation factor within the range of 0 to 1.

- Update process:

According to the standard variational Bayesian rule, there is a posterior probability distribution $p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid \boldsymbol{y}_{1:k})$, i.e.,

$$p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid \boldsymbol{y}_{1:k}) \approx q_x(\boldsymbol{x}_k)q_{\mathrm{R}}(\boldsymbol{R}_k) \ . \tag{6}$$

In (6), $q_x(\boldsymbol{x}_k)$ follows a Gaussian distribution, and $q_{\mathrm{R}}(\boldsymbol{R}_k)$ follows an inverse Wishart partition, they are given by (7).

$$\begin{cases} q_x(\boldsymbol{x}_k) = N(\boldsymbol{x}_{k|k}, \boldsymbol{P}_{k|k}) \\ q_{\mathrm{R}}(\boldsymbol{R}_k) = IW(\boldsymbol{R}_{k|k} \mid v_{k|k}, \boldsymbol{V}_{k|k}) \end{cases} \tag{7}$$

By iteratively calculating and minimizing the Kullback-Leibler (KL) divergence value between the true distribution and the approximate distribution [5], i.e.,

$$\begin{aligned} \mathrm{KL}[q_x(\boldsymbol{x}_k)q_{\mathrm{R}}(\boldsymbol{R}_k) \parallel p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid \boldsymbol{y}_{1:k})] = \\ \int q_x(\boldsymbol{x}_k)\boldsymbol{q}_{\mathrm{R}}(\boldsymbol{R}_k) \times \log\left(\frac{q_x(\boldsymbol{x}_k)q_{\mathrm{R}}(\boldsymbol{R}_k)}{p(\boldsymbol{x}_k, \boldsymbol{R}_k|\boldsymbol{y}_{1:k})}\right)\mathrm{d}\boldsymbol{x}_k\mathrm{d}\boldsymbol{R}_k \end{aligned} \tag{8}$$

and observing noise parameters update as follows,

$$\begin{cases} \boldsymbol{v}_k = \boldsymbol{v}_{k|k-1} + 1 \, , \\ \boldsymbol{V}_{k|k} = \boldsymbol{V}_{k|k-1} + \int \left(\boldsymbol{y}_k - h(\boldsymbol{x}_k)\right)\left(\boldsymbol{y}_k - h(\boldsymbol{x}_k)\right)^{\mathrm{T}} \times \\ N(\boldsymbol{x}_{k|k}, \boldsymbol{P}_{k|k})\mathrm{d}\boldsymbol{x}_k \, . \end{cases} \tag{9}$$

Based on the above steps, we ultimately converge to obtain state vector $\boldsymbol{x}_{k|k}$ and state vector error covariance $\boldsymbol{P}_{k|k}$.

B. Optimization of Variational Bayesian Kalman Filter

During the calculation process of the typical variational Bayesian Kalman filter, it continuously estimates and corrects the predicted values using the observed values, while also continuously estimating and correcting the measurement noise $\boldsymbol{R}$. However, in actual satellite clock bias prediction, the estimation and correction of system noise $\boldsymbol{Q}$ is also very important. To improve the accuracy of the filtering algorithm, this document proposes a combined method of Sage-Husa adaptive Kalman filtering algorithm [6] and variational Bayesian Kalman algorithm to estimate and correct the measurement noise $\boldsymbol{R}$ and system noise $\boldsymbol{Q}$, thereby improving the filtering performance. The overall algorithm is organized as follows:

Based on (1), we can construct the following model，

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{L}_{k-1}\boldsymbol{q}_{k-1} \\ \boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{r}_k \end{cases} \tag{10}$$

where $\boldsymbol{L}_{k-1}$is the system noise driving matrix.

- Prediction process:

The state vector prediction is as follows.

$$\boldsymbol{x}_{k|k-1} = \boldsymbol{F}\boldsymbol{x}_{k-1|k-1} \tag{11}$$

The covariance matrix prediction is as follows.

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}\boldsymbol{P}_{k-1}\boldsymbol{F}^{\mathrm{T}} + \boldsymbol{L}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{L}_{k-1}{}^{T} \tag{12}$$

Predict the observation noise matrix parameters $\boldsymbol{V}$ and $\boldsymbol{v}$, where $0 < |\boldsymbol{G}| \leqslant 1$，$\boldsymbol{G} = \sqrt{\rho}\boldsymbol{I}$ [9] and $0 < \rho < 1$.

$$\begin{cases} \boldsymbol{v}_{k|k-1} = \rho(\boldsymbol{v}_{k-1} - n - 1) + n + 1 \\ \boldsymbol{V}_{k|k-1} = \boldsymbol{G}\boldsymbol{V}_{k-1}\boldsymbol{G}^{\mathrm{T}} \end{cases} \tag{13}$$

- Update process:

$$\begin{cases} \boldsymbol{y}_{k|k-1} = \boldsymbol{H}\boldsymbol{x}_{k|k-1} \\ \boldsymbol{P}_{xy,k|k-1} = \boldsymbol{P}_{k|k-1}\boldsymbol{H}^{\mathrm{T}} \end{cases} \tag{14}$$

As shown in (14), $\boldsymbol{y}_{k|k-1}$ is the observed predicted value, and $\boldsymbol{P}_{xy,k|k-1}$ is the state vector cross-covariance matrix. The initialization of the observation noise parameter is set as follows.

$$\begin{cases} \boldsymbol{v}_{k|k} = \boldsymbol{v}_{k|k-1} + 1 \\ \boldsymbol{V}_{k|k} = \boldsymbol{V}_{k|k-1} \end{cases} \tag{15}$$

1) Loop iteration N times, i=0: N-1:

$$\begin{cases} \boldsymbol{\varepsilon}_k = \boldsymbol{y}_k - \boldsymbol{y}_{k|k-1} \\ \boldsymbol{R}_k^{(i)} = (\boldsymbol{v}_{k|k} - n - 1)^{-1}\boldsymbol{V}_{k|k-1}^{(i)} \\ \boldsymbol{P}_{yy,k|k-1}^{(i+1)} = \boldsymbol{H}\boldsymbol{P}_{k|k-1}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R}_k^{(i)} \end{cases} \tag{16}$$

$$\begin{cases} \boldsymbol{x}_{k|k}^{(i+1)} = \boldsymbol{x}_{k|k-1} + \boldsymbol{K}_k^{(n+1)}\varepsilon_k \\ \boldsymbol{P}_{k|k}^{(i+1)} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k^{(i+1)}\boldsymbol{P}_{yy,k|k-1}^{(i+1)}\left(\boldsymbol{K}_k^{(i+1)}\right)^{\mathrm{T}} \\ \boldsymbol{V}_{k|k}^{(i+1)} = \boldsymbol{V}_{k|k-1} + \boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k{}^{T} + \boldsymbol{H}\boldsymbol{P}_{k|k-1}\boldsymbol{H}^{\mathrm{T}} \end{cases} \tag{17}$$

$$\begin{cases} \boldsymbol{M}_{k-1} = [\boldsymbol{L}_{k-1}^{T}\boldsymbol{L}_{k-1}]^{-1}\boldsymbol{L}_{k-1}^{T} \\ \boldsymbol{Q}_k = (1-d_{k-1})Q_{k-1} + d_{k-1}\left[\boldsymbol{M}_{k-1}\boldsymbol{A}\boldsymbol{M}_{k-1}{}^{T}\right] \\ \boldsymbol{A} = \boldsymbol{K}_k\boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k{}^{T}K_k{}^{T} + P_k - F_{k,k-1}P_{k-1}\mathrm{F}^{T} \end{cases} \tag{18}$$

The above steps can effectively estimate and correct the measurement noise $\boldsymbol{R}$ and system noise $\boldsymbol{Q}$, finally we obtain the $\boldsymbol{x}_{k|k}$, and $\boldsymbol{P}_{k|k}$.In (18), b is that $0.95 \leqslant b \leqslant 0.99$, $d_k = (1-b)/(1-b^k)$, $\boldsymbol{P}_{yy,k|k-1}^{(i+1)}$ is observation error covariance matrix, and the parameters information are detailed in Table 1.

III. Clock Bias Prediction Based on Variational Bayesian Kalman Filtering

A. Clock bias model of spaceborne atomic clocks

The satellite clock bias model can be represented as follows.

$$\boldsymbol{x}(t) = \boldsymbol{a}_0 + \boldsymbol{a}_1 t + 0.5\boldsymbol{a}_2 t^2 + \xi(t) \tag{19}$$

In (19), $\boldsymbol{a}_0$ is the difference between the starting spaceborne atomic clock and the reference clock; $\boldsymbol{a}_1$ is the frequency difference between the starting spaceborne atomic clock and the reference clock; $\boldsymbol{a}_2$ is the aging rate of the onboard atomic clock itself, ξ (t) is the noise, and t is the sampling time [7].

B. Algorithm simulation design

According to (10) and (19), when predicting the clock bias of a spaceborne atomic clock, the superposition of the *k*-th sampled state vector $\boldsymbol{x}_k$ is given by,

$$\boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{L}_{k-1}\boldsymbol{q}_{k-1} \tag{20}$$

$$\boldsymbol{F} = \begin{bmatrix} 1 & t & 0.5t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix} \tag{21}$$

where $\boldsymbol{x}_{k-1} = [\boldsymbol{a}_{0,k-1}, \boldsymbol{a}_{1,k-1}, \boldsymbol{a}_{2,k-1}]^{\mathrm{T}}$, $\boldsymbol{F}$ is the system transmission matrix, and t is the sampling interval, $\boldsymbol{q}_{k-1}$ is a three-dimensional noise that follows a normal distribution with zero mean and is uncorrelated in time, $\boldsymbol{L}_{k-1}$ is the system noise driving matrix.

The clock error measurement value $\boldsymbol{y}_k$ for the k-th sampling is as follow,

$$\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{r}_k \tag{22}$$

where $\boldsymbol{H} = [1,0,0]$ is the measurement matrix， $\boldsymbol{r}_k$ is set as a constant for the initial value of one-dimensional white noise , $\boldsymbol{Q}_k = E(\boldsymbol{q}_k\boldsymbol{q}_k^{\mathrm{T}})$ and $\boldsymbol{R}_k = E(\boldsymbol{r}_k\boldsymbol{r}_k^{\mathrm{T}})$, b is that $0.95 \leqslant b \leqslant 0.99$. In summary, the overall program flow is shown in Table 1:

TABLE I. PROGRAM FLOW

Input: $\boldsymbol{x}_0$、$\boldsymbol{P}_0$、$\boldsymbol{F}$、$\boldsymbol{H}$、$\boldsymbol{t}$、$\boldsymbol{Q}_0$、$\boldsymbol{R}_0$、$\boldsymbol{b}$、$\boldsymbol{L}_0$
Process Predict:
1. State vector predict： $\boldsymbol{x}_{k\|k-1} = \boldsymbol{F}\boldsymbol{x}_{k-1\|k-1}$
2. Covariance matrix predict： $\boldsymbol{P}_{k\|k-1} = \boldsymbol{F}\boldsymbol{P}_{k-1}\boldsymbol{F}^{\mathrm{T}} + \boldsymbol{L}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{L}_{k-1}{}^{T}$
3. Observation noise matrix parameters $\boldsymbol{V}$,$\boldsymbol{v}$: $\boldsymbol{v}_{k\|k-1} = \rho(\boldsymbol{v}_{k-1} - n - 1) + n + 1$ $\boldsymbol{V}_{k\|k-1} = \boldsymbol{G}\boldsymbol{V}_{k-1}\boldsymbol{G}^{\mathrm{T}}$, $0 < \|\boldsymbol{G}\| \leqslant 1$
Measurement update:
1. Observation value update： $\boldsymbol{y}_{k\|k-1} = \boldsymbol{H}\boldsymbol{x}_{k\|k-1}$
2. State vector cross-covariance matrix update： $\boldsymbol{P}_{xy,k\|k-1} = \boldsymbol{P}_{k\|k-1}H^{\mathrm{T}}$,
3. Observation noise parameter settings： $\boldsymbol{v}_{k\|k} = \boldsymbol{v}_{k\|k-1} + 1, \boldsymbol{V}_{k\|k}^{(0)} = \boldsymbol{V}_{k\|k-1}$
Circle *i* = 0 : *N*-1:
1. Observation error calculate： $\boldsymbol{\varepsilon}_k = \boldsymbol{y}_k - \boldsymbol{y}_{k\|k-1}$
2. Observation noise update： $\boldsymbol{R}_k^{(i)} = (\boldsymbol{v}_{k\|k} - n - 1)^{-1}\boldsymbol{V}_{k\|k}^{(i)}$
3. Observation error covariance matrix update： $\boldsymbol{P}_{yy,k\|k-1}^{(i+1)} = \boldsymbol{H}\boldsymbol{P}_{k\|k-1}H^{\mathrm{T}} + \boldsymbol{R}_k^{(i)}$
4. Kalman gain update:$\boldsymbol{K}_k^{(i+1)} = \boldsymbol{P}_{xy,k\|k-1}\left(\boldsymbol{P}_{yy,k\|k-1}^{(i+1)}\right)^{-1}$
5. State vector update： $\boldsymbol{x}_{k\|k}^{(i+1)} = \boldsymbol{x}_{k\|k-1} + \boldsymbol{K}_k^{(n+1)}\varepsilon_k$
6. Covariance matrix update： $\boldsymbol{P}_{k\|k}^{(i+1)} = \boldsymbol{P}_{k\|k-1} - \boldsymbol{K}_k^{(i+1)}\boldsymbol{P}_{yy,k\|k-1}^{(i+1)}\left(\boldsymbol{K}_k^{(i+1)}\right)^{\mathrm{T}}$
7. Observing noise parameters update： $\boldsymbol{V}_{k\|k}^{(i+1)} = \boldsymbol{V}_{k\|k-1} + \boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k{}^{T} + \boldsymbol{H}\boldsymbol{P}_{k\|k-1}\boldsymbol{H}^{\mathrm{T}}$
8. System noise update： $\boldsymbol{M}_{k-1} = [\boldsymbol{L}_{k-1}^{T}\boldsymbol{L}_{k-1}]^{-1}\boldsymbol{L}_{k-1}^{T}$ $\boldsymbol{Q}_k = (1-d_{k-1})\boldsymbol{Q}_{k-1} + d_{k-1}\left[\boldsymbol{M}_{k-1}A\boldsymbol{M}_{k-1}{}^{T}\right]$ $\boldsymbol{A} = \boldsymbol{K}_k\boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k{}^{T}\boldsymbol{K}_k{}^{T} + \boldsymbol{P}_k - \boldsymbol{F}_{k,k-1}\boldsymbol{P}_{k-1}\boldsymbol{F}^{T}$
Output: $\boldsymbol{x}_{k\|k}$、$\boldsymbol{P}_{k\|k}$

IV. Experimental Results and Analysis

In order to verify the effectiveness of the variational Bayesian Kalman filtering algorithm in satellite clock bias prediction, filtering simulation experiments based on classical Kalman filtering and variational Bayesian Kalman filtering algorithms were conducted, as well as satellite clock bias prediction simulation experiments. The experimental results are as follows:

A. Filtering simulation experiment

To verify the performance of the optimized variational Bayesian Kalman filtering algorithm, we selected the SP3 format of precise ephemeris data provided by the French National Center for Space Studies (http://www.pppwizard.net/) for simulation testing. First, we extracted the clock bias data with a sampling interval of 5 minutes from the file and added Gaussian white noise with an average value of zero and a variance of 1 nanosecond to the clock bias data of satellites PRN02, PRN04, PRN06 and PRN08. We used the classical Kalman filtering algorithm and the optimized variational Bayesian Kalman filtering to filter the data of 300 sampling points. In order to better analyze the filtering situation, we selected the root mean square error (RMSE) as a performance indicator to calculate the data.

$$E_{\mathrm{RMSE}} \triangleq \sqrt{\frac{1}{N}\sum_{l=1}^{N}\left(\left(x_k^i-\hat{x}_k^i\right)^2\right)} \tag{23}$$

In (23), x_k^i represents the original satellite clock error, and $\hat{x}_k^i$ represents the satellite clock error after filtering the satellite clock error with additional noise. The simulation results are as follows.

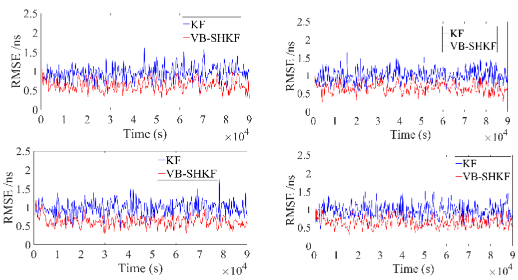

Fig. 1. PRN02, 04, 06, 08 RMSE graph of filtering results

As shown in the Fig.1, under the same initialization parameters, for four sets of satellite clock bias data loaded with Gaussian white noise, the filtering effect of the variational Bayesian Kalman filtering algorithm is better than that of the classical Kalman filtering algorithm. The filtering performance of each satellite has been improved, as shown in Table 2.

TABLE II. THE COMPARISON OF PREDICTION ACCURACY

Algorithm	**RMSE /ns （1500 mins）**			
	PRN02	**PRN04**	**PRN06**	**PRN08**
KF	0.89	0.93	0.90	0.91
VB-SHKF	0.58	0.61	0.59	0.63

B. Clock error prediction simulation experiment

To test the clock bias prediction performance of the optimized variational Bayesian Kalman filter. Based on the 600 original satellite clock bias data with a sampling interval of 5 minutes mentioned above, a prediction experiment is conducted. Take the first 300 satellite clock bias data as training samples and use the last 300 satellite clock bias data as testing samples. During the training process, classic Kalman filtering algorithm and optimized variational Bayesian Kalman filtering algorithm are used to estimate the clock bias state vector, and subsequent prediction of 300 clock biases is achieved.

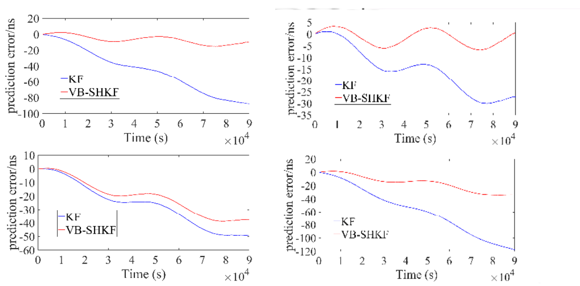

Fig. 2. PRN02, 04, 06, 08 prediction error graph of prediction results

As shown in the Fig.2, under the same initialization parameters, clock bias prediction was performed on the satellite clock bias data of four sets of GPS post precise ephemeris. The clock bias prediction effect of the variational Bayesian Kalman filtering algorithm was improved to a certain extent compared to the classical Kalman filtering algorithm. Calculate the standard deviation for a predicted clock deviation of 1500 minutes, and the results are shown in Table 3.

TABLE III. THE COMPARISONS OF PREDICTION ACCURACY

Algorithm	**Standard Deviation /ns （1500 mins）**			
	PRN02	**PRN04**	**PRN06**	**PRN08**
KF	27.11	9.86	15.87	35.67
VB-SHKF	5.19	3.28	12.57	11.82

V. CONCLUSION

The experiment is based on four sets of GPS precise ephemeris data to conduct filtering simulation experiments on the classical Kalman filtering algorithm and the optimized variational Bayesian Kalman filtering algorithm, as well as satellite clock bias prediction simulation experiments. The experimental results show that the variational Bayesian Kalman filtering algorithm combined with the Sage-Husa filtering algorithm is superior to the classical Kalman filtering to some extent in clock bias filtering and clock bias prediction. This provides a new application idea for clock bias prediction based on Kalman filter.

REFERENCES

[1] Chen, L., Geng, C., Zhou, Q., 2016b.Estimation model and accuracy analysis of BeiDou/GPS real-time precise satellite clock error integrated resolving.Acta Geodaet.Cartogr.Sin.45(9),1028–1034.

[2] Ye, Y. U., Hui-Jun, Z., Xiao-Hui, L. I., Bo, X., & Jing-Ya, C.. (2018). Short-and mid-term prediction of gps satellite clock bias based on GM(1,1) and mecm combination model. Acta Astronomica Sinica.

[3] Chen, R., & Liu, J. S.. (2010). Mixture kalman filters. Journal of the Royal Statistical Society B, 62(3), 493-508.

[4] A. Gelman, J. B. Carlin, H. S. Stern, and D. B. Rubin, Bayesian data analysis. Chapman and Hall/CRC, 1995.

[5] Dong, P. (2019). Adaptive cubature kalman filtering slam algorithm based on variational bayes. Harbin Gongye Daxue Xuebao/Journal of Harbin Institute of Technology, 51(4), 12-18.

[6] Sage A P,Huse G W. Adaptive Filtering with Unknown Prior Statistics. Proceedings of Joint Automatic Control Conference.Boulder;1969.5

[7] Xu, B., Wang, Y., & Yang, X.. (2013). Navigation satellite clock error prediction based on functional network. Neural Processing Letters, 38(2)